OSU LUNAR TWO-FREQUENCY EXPERIMENT

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NOTICES

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by

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Investigation of	Theoretical and Experimental Analysis of the Electromagnetic Scattering and Radiative Properties of Terrain, with Emphasis on Lunar-Like Surfaces
Subject of Report	OSU Lunar Two-Frequency Experiment
Submitted by	W.H. Peake, R. Turpin and R.C. Taylor ElectroScience Laboratory Department of Electrical Engineering
Date	27 July 1967

ABSTRACT

Presented in this report are the results of a two-frequency experiment conducted at The Ohio State University ElectroScience Laboratory. The experimental results are compared with the theoretically predicted behavior in an attempt to verify the fundamental ideas on which the two frequency experiment is based. An estimate is made of the RMS height of the lunar surface.

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OSU LUNAR TWO-FREQUENCY EXPERIMENT

I. INTRODUCTION

Past lunar radar experiments have been predominantly single-frequency experiments. Among the surface parameters estimated by these investigations is the RMS surface slope. It has been shown both experimentally (Fig. 1) and theoretically (Reference 1) that the effective RMS surface slope is a function of the examining wavelength. An estimation of the RMS surface height cannot be obtained from a single-frequency experiment.

Reference 2 describes a "two-frequency experiment" which provides an estimate of the RMS height of a rough surface. Unlike the effective RMS surface slope, the RMS surface height is for all practical purposes independent of the examining wavelength. Estimates of the RMS height and the RMS slope together give a fairly complete picture of the type of rough surface under study.

The results of a two-frequency experiment performed at the ElectroScience Laboratory at The Ohio State University are presented in this report. Instrumentation limitations prevented the performance of the experiment exactly as outlined in Reference 2, but the results should aid in verifying the fundamental ideas on which the two-frequency experiment is based.

II. THE TWO-FREQUENCY EXPERIMENT

The "two-frequency experiment" is described in detail in Reference 2. Basically, a rough surface is illuminated with two waves at two different frequencies f_1 and f_2. The frequency difference is represented by $\Delta f = f_1 - f_2$. The scattered signals are correlated for various values of Δf, providing a correlation coefficient as a function of frequency separation. It is reasonable to expect that for Δf very small, the correlation coefficient will be near unity. When the wavelength corresponding to Δf, i.e., $\lambda_S = c/\Delta f$, (c = velocity of light), becomes of the same order of magnitude as the surface heights, one would expect intuitively that the correlation should begin to decrease. Thus the separation wavelength, λ_S, at which correlation begins to decrease should provide some measure of RMS surface roughness.

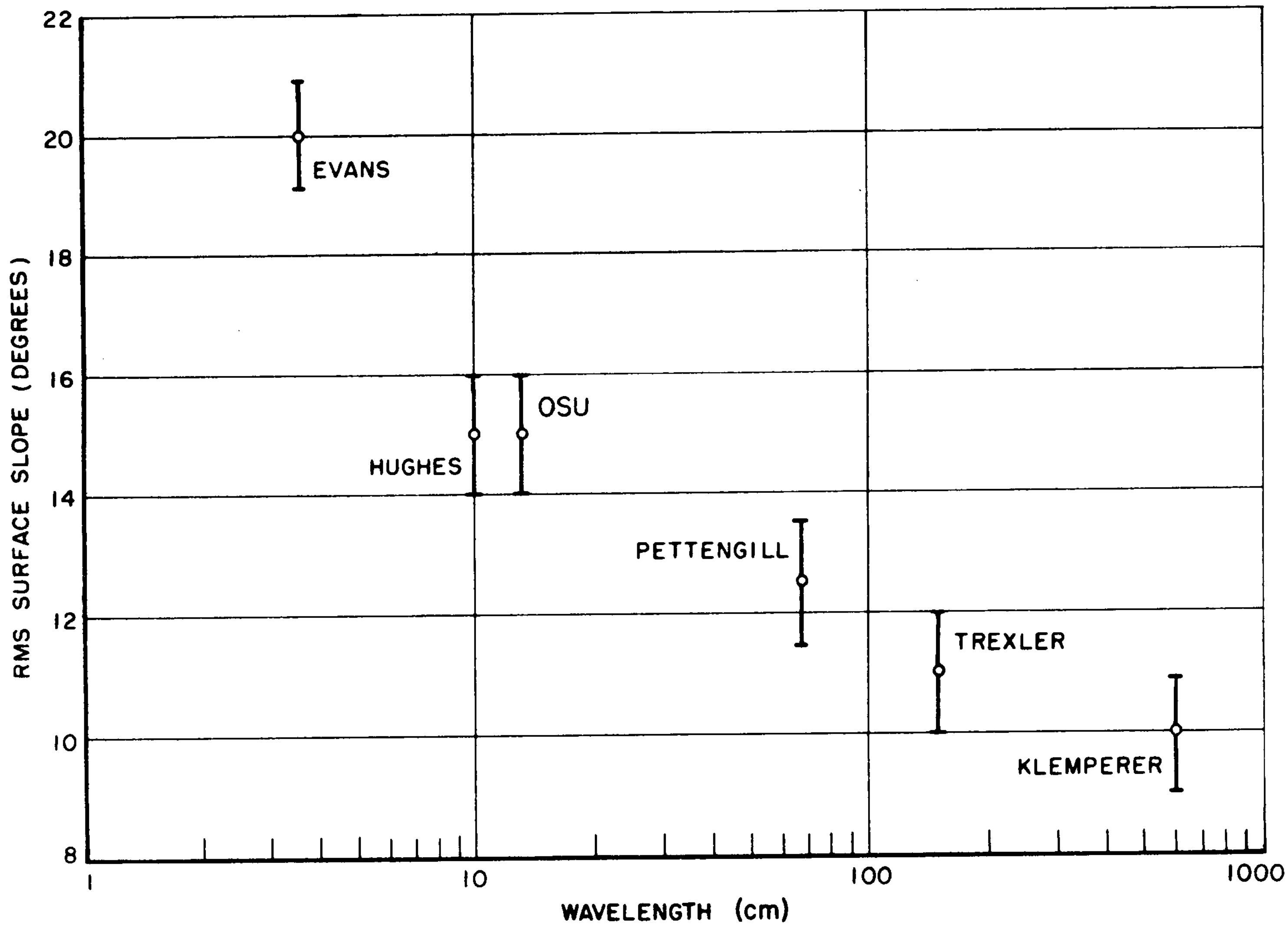

Fig. 1. Frequency dependence of RMS surface slope, derived by comparing experimental[7,8,9] and theoretical[1] backscattering functions.

There is an initial rapid decrease in the correlation coefficient when the separation wavelength is of the order of the diameter of the moon. This, of course, provides no information on the RMS roughness. Previous two-frequency investigations[5] were terminated at this point, before reaching separation frequencies great enough (or separation wavelengths small enough) to detect the region of decrease in the correlation predicted by the two-frequency theory.

For experimental purposes, f_1 and f_2 must be large, e.g., in the microwave range, for convenient measurement. It is also desirable to make f_1 and f_2 large enough for the physical optics approximation to apply for at least the largest scale of surface roughness in order to permit comparison of the theoretical results (which assumed the physical optics approximation) with experimental results. The frequency separation, Δf, can be obtained simply by modulating a carrier. The sidebands of the return signal can then be correlated to provide the desired information.

According to Reference 2, the function of interest in the instance of scattering from a spherical rough surface is

$$(1) \qquad P(\Delta k\sigma) = \sqrt{R\Delta k}\ \mathrm{Cor}(H_1^S H_2^{S*})$$

where $\mathrm{Cor}(H_1^S H_2^{S*})$ is the correlation coefficient of the backscattered fields, i.e., of the sidebands of the return signal, R is the radius of the scattering body, and $\Delta k = 2\pi\Delta f/c$. By noting where the experimentally obtained function $P(\Delta k\sigma)$ falls off to half its initial value (the initial value being where $2R\Delta k$ is still large, e.g., at $\Delta f = 2000$ Hz), an estimate of the surface RMS height can be obtained. It is predicted theoretically in Reference 2 that the correlation coefficient of the backscattered field is

$$(2) \qquad \mathrm{Cor}(H_1^S H_2^{S*}) = \frac{1}{\sqrt{\pi}} \frac{e^{-\frac{1}{S^2}}}{\left[1 - \phi\left(\frac{1}{S}\right)\right]} \frac{1}{\sqrt{R\Delta k}}\ e^{-2\sigma^2 \Delta k^2}$$

where S is the effective RMS surface slope (frequency dependent), K(S) is a function of S and σ is the RMS height. The function $1 - \phi(x)$ is the complement of the error function, $\phi(x)$, and is given by

$$1 - \phi(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2}\, dt \quad .$$

Equation (2) is valid only when $2R\Delta k >> 1$ ($\Delta f > 1000$ Hz in the case of the moon).

The function $P(\Delta k\sigma)$ is called here an enhancement of the correlation coefficient. Figure 2 shows the theoretically predicted curve for $P(\Delta k\sigma)$ assuming a Gaussian JPDF statistical model for the surface. At the point where $P(\Delta k\sigma)$ is one half its initial value it is noted that $2\sigma^2\Delta k^2 = 0.693$. From this it is possible to estimate σ, if the curve is given.

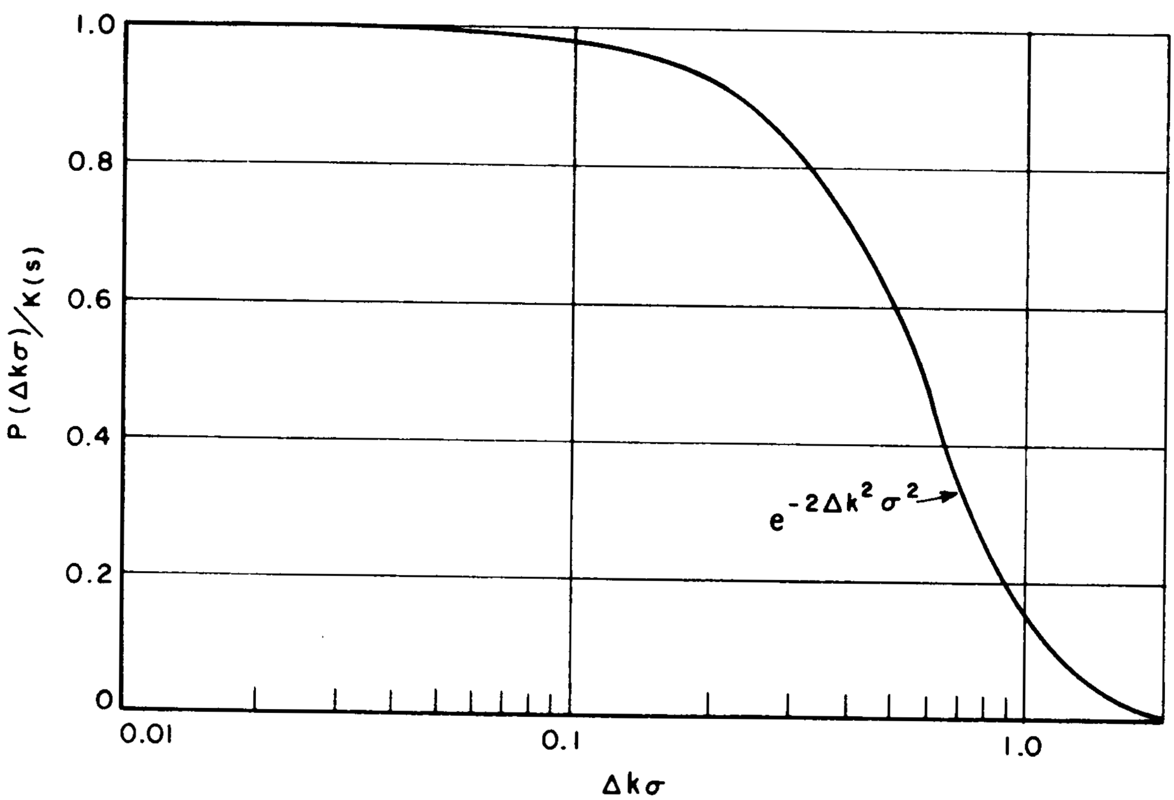

Fig. 2. Theoretical prediction for $P(\Delta k\sigma)$.

III. INSTRUMENTATION

A phase modulated signal was transmitted from The Ohio University (30 ft paraboloidal antenna; 82° 07' 29" west longitude, 39° 19' 28" north latitude). Th carrier frequency was 2270 MHz. The frequency separation, Δf, between the first sidebands ranged from 1 kHz to 60 kHz. The transmitter power was approximately 10 kw.

The receiving station was The Ohio State University Electro-Science Laboratory (82° 02' 30" west longitude, 40° 00' 10" north latitude) using one of an array of four 30 ft paraboloidal antennas. Parametric amplifiers provided approximately 4 dB noise figures, 20 dB gain and 30 MHz bandwidth.

The receiver system was the same as that described in Reference 3 with the exception that two receivers, each with a bandwidth of 2.5 kHz, were added. One receiver was tuned to the upper sideband, the other to the lower sideband. Also, the signals were envelope detected. Figure 3 is a block diagram of the two-frequency receiving system. For a more detailed description see Reference 3.

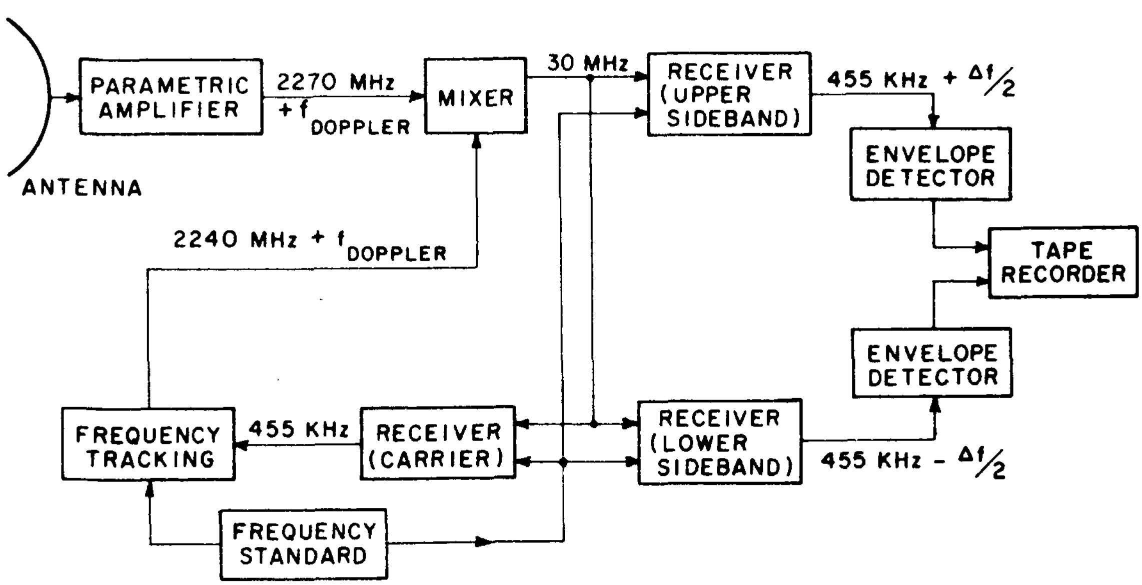

Fig. 3. Two-frequency receiving system.

It must be noted that the correlation according to Reference 2 should be proportional to $E\{H_1^s H_2^{s*}\}$, where $E\{\ \}$ denotes the expected value, H_1^s and H_2^s are the scattered fields at the two frequencies f_1 and f_2, respectively, and $*$ denotes the complex conjugate. The instrumentation for this experiment, however, provides a correlation proportional to $E\{|H_1^s|\,|H_2^s|\}$, thus the phase contribution was lost.

The cross-correlation of two real processes is defined by

$$R_{xy}(t_1, t_2) = E\{x(t_1)\, y(t_2)\} \quad .$$

The cross-covariance is defined by

$$C_{xy}(t_1, t_2) = R_{xy}(t_1, t_2) - m_x(t_1)\, m_y(t_2) \quad ,$$

where $m_x(t) = E\{x(t)\}$ and $m_y(t) = E\{y(t)\}$ define the mean values of $x(t)$ and $y(t)$, respectively. For the purposes of this experiment, $t_1 = t_2$. The processes are assumed stationary.

The correlation coefficient is defined by

$$\rho_{xy} = \frac{C_{xy}}{\sigma_x \sigma_y} = \frac{R_{xy} - m_x m_y}{\sigma_x \sigma_y}$$

where σ_x and σ_y are the standard deviations of $x(t)$ and $y(t)$, respectively, and are given by

$$\sigma_x^2 = E\{(x-m_x)^2\} = E\{x^2\} - m_x^2$$

and

$$\sigma_y^2 = E\{(y-m_y)^2\} = E\{y^2\} - m_y^2 \quad .$$

Thus

(3) $$\rho_{xy} = \frac{E\{xy\} - E\{x\}\, E\{y\}}{\left[E\{x^2\} - E^2\{x\}\right]^{\frac{1}{2}} \left[E\{y^2\} - E^2\{y\}\right]^{\frac{1}{2}}} \quad .$$

Assuming the processes are ergodic, i.e., assuming time averages and ensemble averages are equal, the statistical (ensemble) averages of Eq. (3) can be replaced by time averages to give

$$\rho_{xy} = \lim_{T \to \infty} \frac{\frac{1}{2T}\int_{-T}^{T} x(t)y(t)dt - \left(\frac{1}{2T}\int_{-T}^{T} x(t)dt\right)\left(\frac{1}{2T}\int_{-T}^{T} y(t)dt\right)}{\left[\frac{1}{2T}\int_{-T}^{T} x^2(t)dt - \left(\frac{1}{2T}\int_{-T}^{T} x(t)dt\right)^2\right]^{\frac{1}{2}}\left[\frac{1}{2T}\int_{-T}^{T} y^2(t)dt - \left(\frac{1}{2T}\int_{-T}^{T} y(t)dt\right)^2\right]^{\frac{1}{2}}} \quad .$$

For finite amounts of sampled data, this equation is approximated by

(4) $$\rho_{xy} \approx \frac{\frac{1}{N}\sum_{n=1}^{N} x_n y_n - \left(\frac{1}{N}\right)^2 \sum_{n=1}^{N} x_n \sum_{n=1}^{N} y_n}{\left[\frac{1}{N}\sum_{n=1}^{N} x_n^2 - \left(\frac{1}{N}\sum_{n=1}^{N} x_n\right)^2\right]^{\frac{1}{2}}\left[\frac{1}{N}\sum_{n=1}^{N} y_n^2 - \left(\frac{1}{N}\sum_{n=1}^{N} y_n\right)^2\right]^{\frac{1}{2}}} \quad .$$

By means of this expression estimates of ρ_{xy} as a function of Δf can be obtained. The desired function is then (see Eq. (1), Section II)

(5) $$P_{exp}(\Delta k \sigma) = \sqrt{R \Delta k}\; \rho_{xy}(\Delta f) \quad ,$$

where R is the radius of the moon (1738 km). The computer program used in computing ρ_{xy} is given in the Appendix.

V. EXPERIMENTAL RESULTS

Data were recorded with frequency separation values of $\Delta f = 1, 2, 3, 4, 8, 20, 40, 60$ kHz. The length of each recording period was about 10 minutes. The data were recorded on magnetic tape. Samples of both the upper and the lower sideband data for frequency separations of 1 kHz and 60 kHz are shown in Fig. 4.

Analog-to-digital conversion was performed at the OSU Computer Center. The data were sampled at a rate of 125 samples per second per channel.

Estimates of the correlation function, $P_{exp}(\Delta k\sigma)$, were computed from the available data according to Eqs. (4) and (5) of Section IV. The results are shown in Fig. 5. The experimental results can be compared with the theoretical curve of Fig. 2 after normalizing $P_{exp}(\Delta k\sigma)$ with respect to an "initial value" of 0.84 (derived from Fig. 5). This comparison is shown in Fig. 6, where the "predicted" curve has been matched to the experimental points to provide the best fit. It is seen that the normalized, enhanced correlation function reaches half its initial value at a frequency separation of about 60 kHz, which corresponds, according to the relation[2]

$$2\sigma^2 \Delta k^2 = 0.693, \tag{6}$$

to an RMS surface height, σ, of about 500 meters. This can be compared with an estimate[6] of about 1.4 kilometers obtained from the contour map of the limb region that was constructed by Hayn (1914) from photographs of the lunar profile.

Due to the small amount of data available for each value of Δf, no meaningful ranges of experimental error can be attached to the points obtained. It is suggested[4, 5] in theory that the error in estimation of the correlation coefficient is inversely proportional to the square root of the total number of independent pairs of sample points. Possible error limits on this basis are provided in Figs. 5 and 6. They are purely statistical in nature.

No definite explanation can be given for the discrepancy of the point at $2\Delta f = 4$ kHz. The presence of an additional noise component could have produced the decrease in correlation.

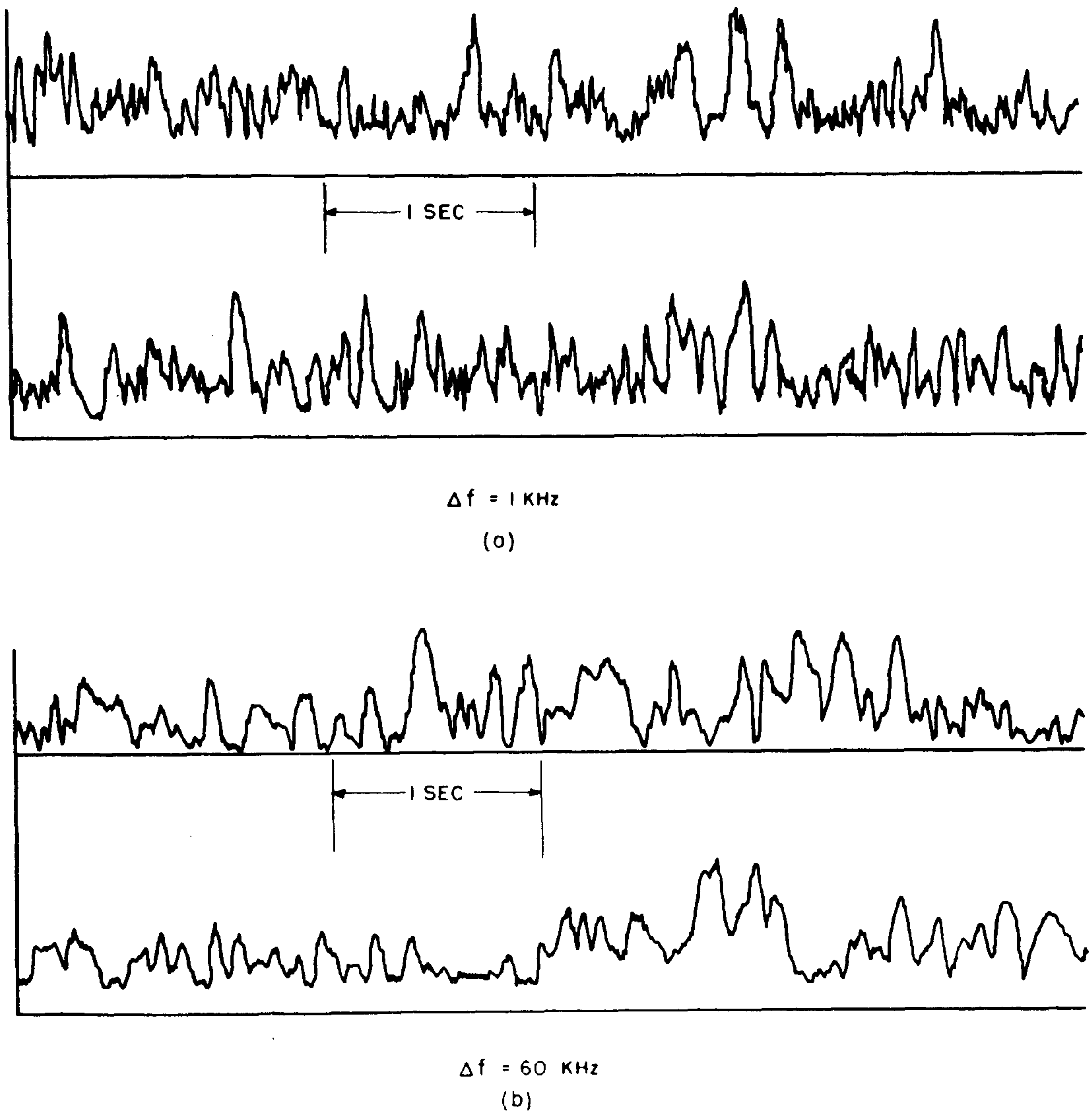

Fig. 4. Upper and lower sidebands of return signals for Δf = 1 kHz and Δf = 60 kHz.

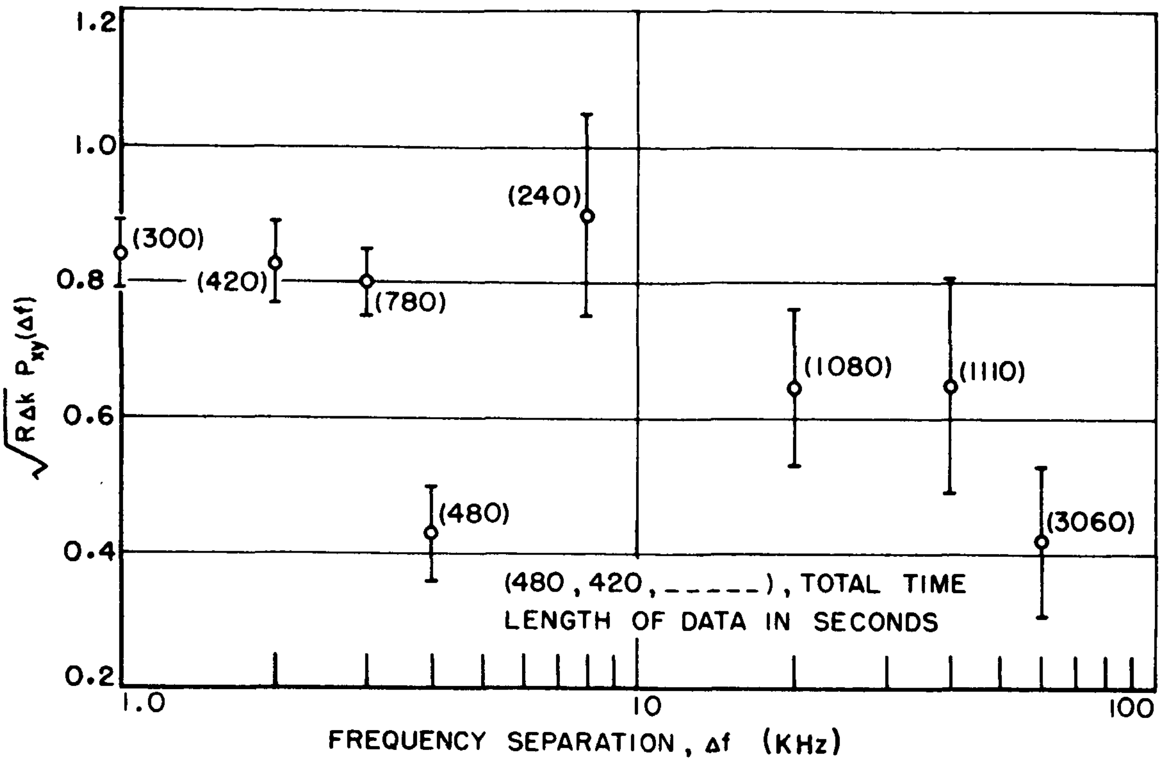

Fig. 5. "Enhanced" correlation function, $P_{exp}(\Delta k\sigma)$.

VI. CONCLUSIONS

Even though the experiment discussed in the preceding sections did not comply exactly with the two-frequency experiment described in Reference 2, it is felt, in light of the results obtained, that certain conclusions can be drawn. First, the experimental results of Section V suggest that the RMS height of the visible lunar surface is on the order of 500 meters. Secondly, the general agreement of the experimental enhanced correlation function (Fig. 6) with the theoretically predicted behavior tends to verify the theoretical basis of the two-frequency experiment. Finally, it is concluded that these results have shown the two-frequency experiment to be a valid technique to be used, along with established methods, in the study of remote surfaces, and hence merits further development.

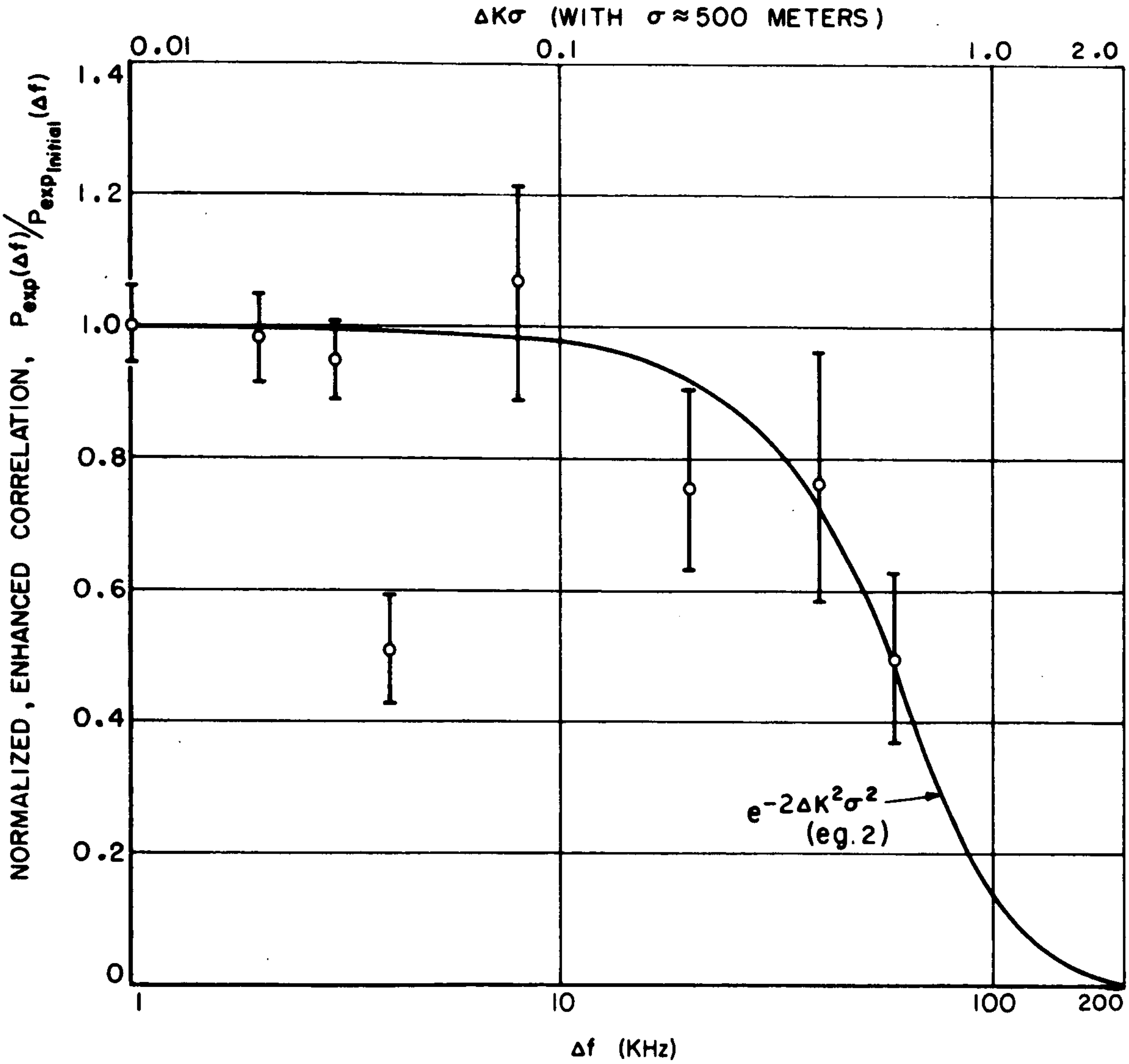

Fig. 6. Normalized, enhanced correlation function compared with theoretical predictions.

VII. RECOMMENDATIONS FOR FUTURE WORK

The "two-frequency experiment" described in Reference 2 and the supporting experimental results presented in preceding sections of this report represent a significant contribution to the understanding of the problem of studying planetary surfaces by radar methods. A repetition of the two-frequency experiment is suggested, however, with equipment suitable for predetection correlation measurements, over a larger range of frequency separation (out to, say, $\Delta f = 120$ KHz). It would be desirable to use a carrier frequency no higher than a few GHz, since it is not known by how much the noise due to the diffuse component of the lunar scattering may reduce the observable correlation between the two frequency components.

Further theoretical development of the two-frequency method (i.e., to include post detection correlation experiments, non-physical optics surface models, etc.) could also be profitably undertaken.

ACKNOWLEDGEMENTS

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APPENDIX
SCATRAN COMPUTER PROGRAM FOR COMPUTING ρ_{xy}

Description of input and output terms:

NTAPE	=	Digital tape identification
NRUNS	=	Number of sets of data to be correlated
IFSEP	=	Frequency separation for current set of data
NSPS	=	Data sample rate
NWR	=	Number of words per record
NCHS	=	Number of channels on the tape
NCH1, NCH2	=	Numbers of channels to be correlated
NFSKIP	=	Number of files skipped to reach data
NOBACK	=	If not equal to zero, will backspace a number of records equal to the number of records just read
NOPLOT	=	If not equal to zero, will plot data by means of the Scatran plot package
NS	=	Every NSth data point is used in computing the correlation
TSKIP	=	Time in seconds to skip within a file to reach data
TCOR	=	Time length of data (in seconds) to correlate
KXY	=	Correlation coefficient
XMEAN, YMEAN	=	Mean value of each channel
XDEV, YDEV	=	Standard deviation of each channel

```
 **    INPUT      * B01       I HB                           TAPE NO. 605
 ***   RUN,DUMPLOWERCORE,SCATRAN
C                -
C              TWO-FREQUENCY CORRELATION COEFFICIENT-
C                -
               DIMENSION(FMT(12),IX(1000),IY(1000))-
               FLOATING(KXY)-
  START        READ INPUT,8,(NTAPE)-
               WRITE OUTPUT,FHEAD,(NTAPE)-
F FHEAD        (38H TWO-FREQUENCY CORRELATION COEFFICIENT,10X,9HTAPE NO. ,
                    I3////)-
               DEFINE POOL,POOL,2,501-
  FLIST        FILE LIST(A,$INPUT $)-
               ATTACH FILES,POOL,A,1-
               NFILE=1-
               READ INPUT,8,(NRUNS)-
               DO THROUGH(END),NUR=1,1,NUR.LE.NRUNS-
               READ INPUT,8,(IFSEP,NSPS,NWR,NCHS,NCH1,NCH2,NFSKIP,NOBACK,
                    NOPLOT,NS)-
               NFILE=NFILE+NFSKIP-
               READ INPUT,FORMAT,((FMT(I),I=0,1,I.L.12))-
F FORMAT       (12L6)-
               PROVIDED(NOBACK.E.0),TRANSFER TO (TSTC)-
               WRITE OUTPUT,FBACK,(TCOR)-
F FBACK        (16H         BACK UP ,F5.1,5H SEC./)-
  TSTC         READ INPUT,7,(TSKIP,TCOR)-
               DO THROUGH(SKIPFI),NFI=0,1,NFI.L.NFSKIP-
  READ         READ DECIMAL,A,SKIPFI,9-
               TRANSFER(READ)-
  SKIPFI       CONTINUE-
               NRSKIP=TSKIP*NSPS*NCHS/(3*NWR)+.5-
               DO THROUGH(SKIP),I=1,1,I.LE.NRSKIP-
  SKIP         READ DECIMAL,A,EOF,9-
               PROVIDED(NOBACK.E.0),TRANSFER TO (NOPE)-
C              TO CORRELATE DIFFERENT CHANNELS BUT SAME TIME AS PREVIOUS RU
               N------
               BACKSPACE RECORDS,A,NOPE,NRDATA-
  NOPE         NRDATA=TCOR*NSPS*NCHS/(3*NWR)+.5-
               WRITE OUTPUT,FFRUN,(NUR,NSPS,NWR,NCH1,NCH2,NCHS,NFILE,TSKIP,
                    TCOR,NS)-
F FFRUN        (10H  RUN NO. ,I2//5X,I5,17H SAMPLES/SEC.     ,I5,
                    13HWORDS/RECORD /5X,13HCORRELATE CH.,I2,8H AND CH.,I2,
                    4H OF ,I2,22H CHS.,        FILE NO. ,I1//5X,5HSKIP ,
                    F5.1,17H SEC., CORRELATE ,F5.1,12H SEC., NS = ,I1//)-
               NJ=3*NWR/NCHS-
               MXY=0-
               MX=0-
               MY=0-
               MXX=0-
               MYY=0-
               DO THROUGH(WORK),NR=1,1,NR.LE.NRDATA-
               READ DECIMAL,A,EOF,FMT,(IA,IB,IC,ID,(IX(J),IY(J),J=1,1,
                    J.LE.NJ))-
               PROVIDED(NOPLOT.E.0),TRANSFERTO(CORR)-
               CALL SUBROUTINE()=SYMBOL.(0.,0.,.07,1,0.,-1)-
               CALL SUBROUTINE()=PLOT.(0.,.001*IX(1),3)-
               DO THROUGH(PLOTX),J=1,NS,J.L.NJ-
  PLOTX        CALL SUBROUTINE()=PLOT.(J*.01575   ,.001*IX(J+1),2)-
               CALL SUBROUTINE()=SYMBOL.(0.,15.,.07,1,0.,-1)-
               CALL SUBROUTINE()=PLOT.(0.,15.+.001*IY(1),3)-
               DO THROUGH(PLOTY),J=1,NS,J.L.NJ-
  PLOTY        CALL SUBROUTINE()=PLOT.(J*.01575   ,.001*IY(J+1)+15.,2)-
               CALL SUBROUTINE()=PLOT.(NJ      *.01575,0.,-3)-
  CORR         DO THROUGH(WORK),J=1,NS,J.LE.NJ-
```

```
              MXY=MXY+IX(J)*IY(J)-
              MX=MX+IX(J)-
              MY=MY+IY(J)-
              MXX=MXX+IX(J)*IX(J)-
  WORK        MYY=MYY+IY(J)*IY(J)-
              TRANSFER(IGNORE)-
  EOF         WRITE OUTPUT,FEOF,(NR)-
F FEOF        (12H END OF FILE,5X,5HNR = ,I3)-
  IGNORE      N=NJ*(NR-1)/NS-
              XMEAN=1.*MX/N-
              YMEAN=1.*MY/N-
              XVAR=1.*MXX/N-XMEAN.P.2-
              YVAR=1.*MYY/N-YMEAN.P.2-
              KXY=(1.*MXY/N-XMEAN*YMEAN)/SQRT.(XVAR*YVAR)-
  END         WRITE OUTPUT,FOUT,(IFSEP,KXY,XMEAN,YMEAN,SQRT.(XVAR),
                    SQRT.(YVAR))-
F FOUT        (8H       K(I2,7HKHZ) = ,F10.5,4X,8HXMEAN = ,F8.3,4X,
                    8HYMEAN = ,F8.3,4X,7HXDEV = ,F8.3,4X,7HYDEV = ,F8.3//)-
              PROVIDED(NOPLOT.NE.0),CALL SUBROUTINE()=PLOTE.()-
              CLOSE UNLOAD,A,1-
              CALL SUBROUTINE()=ENDJOB.()-
              ENDPROGRAM(START)-
 ***   DATA
  605
     1
   40   125   500     2     1     2     2     0     0     1
(N18,3N6,750(C12,C12))
        0.        120.
```